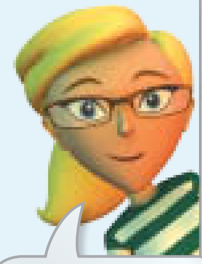
2-2 Conditional Statements

Common Core State Standards

Prepares for G-CO.C.9 Prove theorems about lines and angles. **Also Prepares for G-CO.C.10, G-CO.C.11**

MP 3, MP 6, MP 7

Objectives To recognize conditional statements and their parts
To write converses, inverses, and contrapositives of conditionals



Try to make sense of the statements on the bumper stickers by figuring out when they are true, and when they are false.

MATHEMATICAL PRACTICES

Solve It! Getting Ready!

The company that prints the bumper sticker at the left below accidentally reworded the original statement and printed the sticker three different ways. Suppose the original bumper sticker is true. Are the other bumper stickers true or false? Explain.

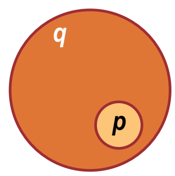
The study of *if-then* statements and their truth values is a foundation of reasoning.

Essential Understanding You can describe some mathematical relationships using a variety of *if-then* statements.

Lesson Vocabulary
- conditional
- hypothesis
- conclusion
- truth value
- negation
- converse
- inverse
- contrapositive
- equivalent statements

take note

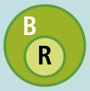
Key Concept Conditional Statements

Definition	Symbols	Diagram
A **conditional** is an *if-then* statement. The **hypothesis** is the part p following *if*. The **conclusion** is the part q following *then*.	$p \rightarrow q$ Read as "if p then q" or "p implies q."	

The Venn diagram above illustrates how the set of things that satisfy the hypothesis lies inside the set of things that satisfy the conclusion.

Think

What would a Venn diagram look like?
A robin is a kind of bird, so the set of robins (R) should be inside the set of birds (B).



Problem 1 Identifying the Hypothesis and the Conclusion

What are the hypothesis and the conclusion of the conditional?
If an animal is a robin, then the animal is a bird.

Hypothesis (p): An animal is a robin.

Conclusion (q): The animal is a bird.

Got It? **1.** What are the hypothesis and the conclusion of the conditional?
If an angle measures 130, then the angle is obtuse.

Think

Which part of the statement is the hypothesis (p)?
For two angles to be vertical, they must share a vertex. So the set of vertical angles (p) is inside the set of angles that share a vertex (q).

Problem 2 Writing a Conditional

How can you write the following statement as a conditional?
Vertical angles share a vertex.

Step 1 Identify the hypothesis and the conclusion.

Vertical angles share a vertex.

Step 2 Write the conditional.

If two angles are vertical, then they share a vertex.

Got It? **2.** How can you write "Dolphins are mammals" as a conditional?

The **truth value** of a conditional is either *true* or *false*. To show that a conditional is true, show that every time the hypothesis is true, the conclusion is also true. A counterexample can help you determine whether a conditional with a true hypothesis is true. To show that the conditional is false, if you find one counterexample for which the hypothesis is true and the conclusion is false, then the truth value of the conditional is false.

Plan

How do you find a counterexample?
Find an example where the hypothesis is true, but the conclusion is false. For part (B), find a number divisible by 3 that is not odd.

Problem 3 Finding the Truth Value of a Conditional

Is the conditional *true* or *false*? If it is false, find a counterexample.

A **If a woman is Hungarian, then she is European.**

The conditional is true. Hungary is a European nation, so Hungarians are European.

B **If a number is divisible by 3, then it is odd.**

The conditional is false. The number 12 is divisible by 3, but it is not odd.

Got It? **3.** Is the conditional *true* or *false*? If it is false, find a counterexample.

a. If a month has 28 days, then it is February.

b. If two angles form a linear pair, then they are supplementary.

The **negation** of a statement p is the opposite of the statement. The symbol is $\sim p$ and is read "not p." The negation of the statement "The sky is blue" is "The sky is *not* blue." You can use negations to write statements related to a conditional. Every conditional has three related conditional statements.

take note

Key Concept Related Conditional Statements

Statement	How to Write It	Example	Symbols	How to Read It
Conditional	Use the given hypothesis and conclusion.	If $m\angle A = 15$, then $\angle A$ is acute.	$p \rightarrow q$	If p, then q.
Converse	Exchange the hypothesis and the conclusion.	If $\angle A$ is acute, then $m\angle A = 15$.	$q \rightarrow p$	If q, then p.
Inverse	Negate both the hypothesis and the conclusion of the conditional.	If $m\angle A \neq 15$, then $\angle A$ is not acute.	$\sim p \rightarrow \sim q$	If not p, then not q.
Contrapositive	Negate both the hypothesis and the conclusion of the converse.	If $\angle A$ is not acute, then $m\angle A \neq 15$.	$\sim q \rightarrow \sim p$	If not q, then not p.

Below are the truth values of the related statements above. **Equivalent statements** have the same truth value.

Statement	Example	Truth Value
Conditional	If $m\angle A = 15$, then $\angle A$ is acute.	True
Converse	If $\angle A$ is acute, then $m\angle A = 15$.	False
Inverse	If $m\angle A \neq 15$, then $\angle A$ is not acute.	False
Contrapositive	If $\angle A$ is not acute, then $m\angle A \neq 15$.	True

A conditional and its contrapositive are equivalent statements. They are either both true or both false. The converse and inverse of a statement are also equivalent statements.

Problem 4 Writing and Finding Truth Values of Statements

What are the converse, inverse, and contrapositive of the following conditional? What are the truth values of each? If a statement is false, give a counterexample.

If the figure is a square, then the figure is a quadrilateral.

Think

Identify the hypothesis and the conclusion.

Write

p: The figure is a square.

q: The figure is a quadrilateral.

Think

To write the converse, switch the hypothesis and the conclusion. Write $q \rightarrow p$.

Write

Converse: If the figure is a quadrilateral, then the figure is a square.

The converse is false.

Counterexample:

A rectangle that is not a square.

Think

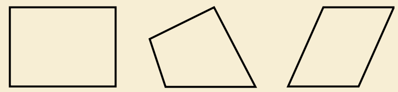
To write the inverse, negate both the hypothesis and the conclusion of the conditional. Write $\sim p \rightarrow \sim q$.

Write

Inverse: If the figure is not a square, then the figure is not a quadrilateral.

The inverse is false.

Counterexamples:



Think

To write the contrapositive, negate both the hypothesis and the conclusion of the converse. Write $\sim q \rightarrow \sim p$.

Write

Contrapositive: If the figure is not a quadrilateral, then the figure is not a square.

The contrapositive is true.

Got It? **4.** What are the converse, inverse, and contrapositive of the conditional statement below? What are the truth values of each? If a statement is false, give a counterexample.

If a vegetable is a carrot, then it contains beta carotene.

Lesson Check

Do you know HOW?

1. What are the hypothesis and the conclusion of the following statement? Write it as a conditional.

Residents of Key West live in Florida.

2. What are the converse, inverse, and contrapositive of the statement? Which statements are true?

If a figure is a rectangle with sides 2 cm and 3 cm, then it has a perimeter of 10 cm.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. Error Analysis Your classmate rewrote the statement "You jog every Sunday" as the following conditional. What is your classmate's error? Correct it.

If you jog, then it is Sunday.

4. Reasoning Suppose a conditional statement and its converse are both true. What are the truth values of the contrapositive and inverse? How do you know?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Identify the hypothesis and conclusion of each conditional. See Problem 1.

5. If you are an American citizen, then you have the right to vote.

6. If a figure is a rectangle, then it has four sides.

7. If you want to be healthy, then you should eat vegetables.

Write each sentence as a conditional. See Problem 2.

8. Hank Aaron broke Babe Ruth's home-run record.

9. **Algebra** $3x - 7 = 14$ implies that $3x = 21$.

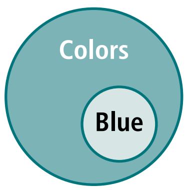
10. Thanksgiving in the United States falls on the fourth Thursday of November.

11. A counterexample shows that a conjecture is false.

12. **Coordinate Geometry** A point in the first quadrant has two positive coordinates.

Write a conditional statement that each Venn diagram illustrates.

13.



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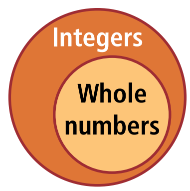

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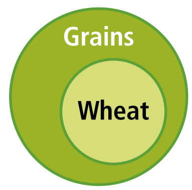


Determine if the conditional is *true* or *false*. If it is false, find a counterexample. See Problem 3.

16. If a polygon has eight sides, then it is an octagon.

17. If you live in a country that borders the United States, then you live in Canada.

18. If you play a sport with a ball and a bat, then you play baseball.

19. If an angle measures 80, then it is acute.

If the given statement is not in *if-then* form, rewrite it. Write the converse, inverse, and contrapositive of the given conditional statement. Determine the truth value of all four statements. If a statement is false, give a counterexample. See Problem 4.

20. If you are a quarterback, then you play football.

21. Pianists are musicians.

22. **Algebra** If $4x + 8 = 28$, then $x = 5$.

23. Odd natural numbers less than 8 are prime.

24. Two lines that lie in the same plane are coplanar.

B Apply

Write each statement as a conditional.

25. "We're half the people; we should be half the Congress." —Jeanette Rankin, former U.S. congresswoman, calling for more women in office

26. "Anyone who has never made a mistake has never tried anything new." —Albert Einstein

27. Probability An event with probability 1 is certain to occur.

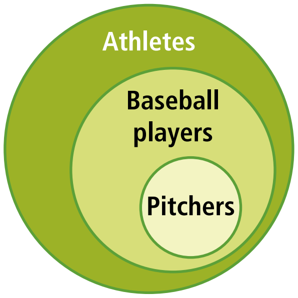
28. Think About a Plan Your classmate claims that the conditional and contrapositive of the following statement are both true. Is he correct? Explain.

If $x = 2$, then $x^2 = 4$.

- Can you find a counterexample of the conditional?
- Do you need to find a counterexample of the contrapositive to know its truth value?

29. Open-Ended Write a true conditional that has a true converse, and write a true conditional that has a false converse.

30. Multiple Representations Write three separate conditional statements that the Venn diagram illustrates.



31. Error Analysis A given conditional is true. Natalie claims its contrapositive is also true. Sean claims its contrapositive is false. Who is correct and how do you know?

Draw a Venn diagram to illustrate each statement.

32. If an angle measures 100, then it is obtuse.

33. If you are the captain of your team, then you are a junior or senior.

34. Peace Corps volunteers want to help other people.

Algebra Write the converse of each statement. If the converse is true, write *true*. If it is not true, provide a counterexample.

35. If $x = -6$, then $|x| = 6$.

36. If y is negative, then $-y$ is positive.

37. If $x < 0$, then $x^3 < 0$.

38. If $x < 0$, then $x^2 > 0$.

39. Advertising Advertisements often suggest conditional statements. What conditional does the ad at the right imply?

Write each postulate as a conditional statement.

40. Two intersecting lines meet in exactly one point.

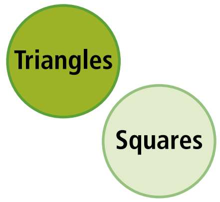
41. Two congruent figures have equal areas.

42. Through any two points there is exactly one line.

Challenge **Write a statement beginning with *all*, *some*, or *no* to match each Venn diagram.**

43.

44.



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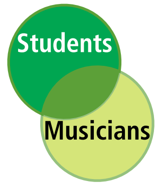

46. Let a represent an integer. Consider the five statements r, s, t, u, and v.

r: a is even. s: a is odd. t: $2a$ is even. u: $2a$ is odd. v: $2a + 1$ is odd.

How many statements of the form $p \rightarrow q$ can you make from these statements? Decide which are true, and provide a counterexample if they are false.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

In the Apply What You've Learned in Lesson 2-1, you made a conjecture based on patterns you observed in the numbers on the calendar page shown on page 81. Your conjecture may have been similar to the one below.

Conjecture: The sums of the numbers on the diagonals of a 2-by-2 calendar square are equal.

You can express this conjecture with symbols by using a, b, c, and d to represent any four distinct numbers from a monthly calendar page. Assume the list a, b, c, d gives the numbers in order from least to greatest.

a. Which of the following conditional statements represents the conjecture? What is the hypothesis of the conditional? What is the conclusion?

I. If $a + d = b + c$, then a, b, c, and d form a 2-by-2 calendar square.

II. If a, b, c, and d form a 2-by-2 calendar square, then $a + d = b + c$.

b. Identify the other statement in part (a) as the *converse*, *inverse*, or *contrapositive* of the conjecture. Then write the other two of these (converse, inverse, or contrapositive) in if-then form.

c. You now have four conditional statements about distinct numbers a, b, c, and d from a monthly calendar page. For which of these statements can you find counterexamples? If possible, give at least two counterexamples.